Estimating long-term implied volatility for the valuation of insurance liabilities

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Market-consistent economic scenarios are at the core of the valuation of liabilities under Solvency II and certain risk-based capital (RBC) regimes in Asia, the valuation of liabilities under IFRS 17 and the valuation of market risk benefits under Long-Duration Targeted Improvements (LDTI). As spot market data is a key input of such valuation frameworks, stabilising the market-consistent valuation of liabilities is increasingly perceived as a material challenge for insurance companies, with a particular focus on how to derive market-consistent but stable long-term volatility assumptions. This paper explores a few potential options available to insurance companies.

To project interest rates or equity returns under a risk-neutral and market-consistent framework, different market-consistent financial models can be used with an objective to match as closely as possible the actual market price of the observable financial instruments. The calibration process is typically based on minimising gaps between the actual observable market prices of derivatives and the prices implied by the financial model (so-called model prices). Examples of interest rate models range from the simplest ones, e.g., one-factor or two-factor short rate models, to the most complex ones, e.g., the LIBOR Market Model with Stochastic Volatility and Displaced Diffusion (DDSVLMM). Examples of equity models typically used in practice include the Black-Scholes model, the Heston model and the Stochastic Volatility Jump Diffusion (SVJD) model.[2]

One of the key strengths of market-consistent models is based on their ability to replicate observable market prices for a wide range of tenors representative of the liability characteristics of insurance companies. This requirement is typically clearly stated under various valuation and capital frameworks, e.g., International Financial Reporting Standard (IFRS) 17, LDTI and Solvency II, as illustrated below for Solvency II:

> "Insurance and reinsurance undertakings should be able to demonstrate that the choice of financial instruments used in the calibration process is relevant given the characteristics of [their] obligations."[3]

In particular, the above requirement emphasises the long-term nature of policyholders' benefits, and the need for financial models to reflect such characteristics, by ensuring long-term derivatives can effectively be taken into account in the calibration process of financial models. That being said, as longer-term derivatives are generally associated with fewer trades, the access to long-term reliable financial data is typically more challenging so that the calibration of financial model would focus more on liquid financial data. Such focus is highlighted by regulators as shown below in the case of Solvency II:

> "Insurance and reinsurance undertakings should ensure that the calibration process of an ESG used for a market consistent valuation is based on data from financial markets that are deep, liquid and transparent".[4]

The two above requirements appear contradictory, although in practice the relevant trade-off needs to be assessed by insurance companies between (1) reflecting the characteristics of the liability, and (2) ensuring a sufficient level of liquidity of the underlying financial instruments to be used for the calibration of the financial model.

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[2] A wide range of financial models is available in the economic scenario generator Milliman CHESS™, including DDSVLMM and SVJD. Please refer to the following website for more details: https://www.milliman.com/en/products/milliman-chess.

[3] European Insurance and Occupational Pensions Authority (EIOPA) Guidelines on the valuation of technical provisions.

In this paper, we focus on the challenge of estimating equity-implied volatility surfaces in the context of the trade-off as mentioned above. In particular, in the following sections, after giving a few elements of context on the derivative market, a possible extrapolation approach to calibrate long-term implied volatilities is described together with its limitations. Subsequently, an IHS Markit (now a part of S&P Global) alternative methodology is introduced. In the last section, an impact study on relevant capital-related indicators is then provided together with a few key takeaways. It is worth noticing that, although the example of the equity-implied volatility has been chosen, most of the results below would also be valid for other instruments and asset classes.

The calibration process

A financial model is simplistically a function of parameters which need to be calibrated in a way such that model outputs can match as closely as possible actual market data observed. A key input of the calibration process is therefore a set of observable data that might potentially be adjusted within the model. Process-wise, it can be split into the following steps:

- **Step 1**: Gathering relevant observable market data representative of the risk factor to be modelled (e.g., equity risk via the implied volatility surface of equity options) and representative of the underlying characteristics of the insurance portfolio.
- **Step 2**: Feed the market data into the financial model.
- **Step 3**: Transform the market data input (to the extent needed) as per the technical specifications of the financial model to produce risk-neutral simulations.

Step 1 is a critical step of the process. The level accuracy of the final outcome can only be as good as the inputs used to calibrate the model. It is therefore crucial to carefully source the market data.

The derivatives market

In order to gather relevant data to ensure Step 1 of the calibration process as outlined above, one relies on market information about equity derivatives. Such derivatives are traded in two ways, namely the listed markets or the over-the-counter (OTC) markets. Different types of information can be collected in both markets as highlighted below.

LISTED MARKETS

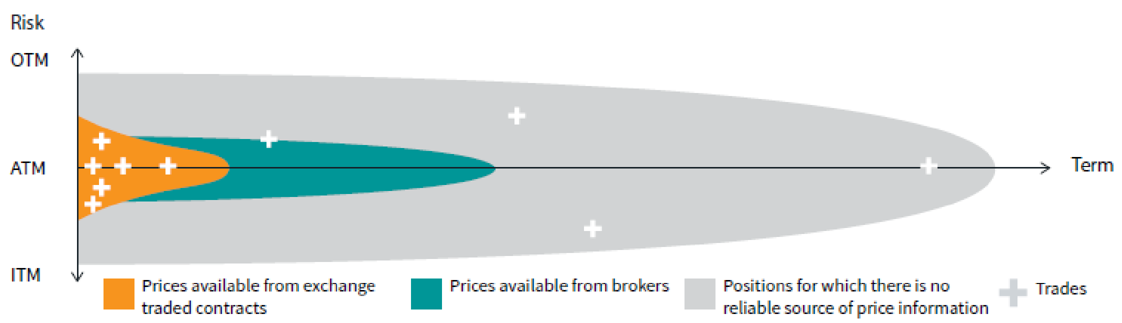
Listed markets are organised markets where traded instruments are standardised. Market is made by the exchange. Some market data such as quotations (pre-trade), settlement prices or traded volume information can be sourced from the various exchanges.

OTC MARKETS

By opposition to the listed market, the OTC market is decentralised, and traded instruments are bespoke. Market is made by dealers. Within OTC markets, information is scattered and scarce. Part of that information is seen by brokers. There are however a variety of brokers, which makes the information more difficult to summarise.

In Figure 1 below, an implied volatility surface with short-term and long-term maturities as well as at-the-money (ATM), in-the-money (ITM) and out-of-the-money (OTM) strikes is shown. In particular, the area in amber represents the part of the volatility surface that can be sourced using listed market data, i.e., mainly for short-term maturities and around the ATM strikes. The area in green represents the part of the volatility surface where brokers may provide additional information, i.e., mostly around the ATM strikes where most trading happens.

FIGURE 1: DATA STRUCTURE FOR OPTION-IMPLIED VOLATILITIES



The challenge is therefore to find solutions to provide accurate data in the area represented in grey.

Limitations of extrapolation techniques

A possible approach to deriving long-term volatility data relies on extrapolation techniques. Although these approaches can appear appealing at first sight, as they allow recovery of full-term structures while overcoming data limitations for longer terms, they can be very sensitive to a few data points and therefore can be volatile from one valuation date to another.

To illustrate those limitations, we consider below one approach often used to extrapolate volatility data points. Such an approach assumes that the instantaneous variance for each strike K beyond the last observed point remains constant. Mathematically, the constant variance extrapolation can be split into three steps:

- **Step 1**: Compute the instantaneous variance as:

$$Var_{inst} = \frac{Var_{T_N}(K) - Var_{T_{N-1}}(K)}{T_N - T_{N-1}},$$

  where the implied volatility for strike K at the two last maturity points available T_{N-1} and T_N are:

$$Var_{T_N}(K) = T_N \sigma_N^2(K)$$

$$Var_{T_{N-1}}(K) = T_{N-1} \sigma_{N-1}^2(K)$$

- **Step 2**: Derive the extrapolated instantaneous variance for any future term $T > T_N$ as:

$$Var_T(K) = Var_{T_N}(K) + (T - T_N) Var_{inst}$$

- **Step 3**: Deduce the long-term implied volatility as follows:

$$\widehat{\sigma_T(K)} = \sqrt{\frac{Var_T(K)}{T}}.$$

Figure 2 illustrates the above methodology.

FIGURE 2: CONSTANT INSTANTANEOUS VARIANCE EXTRAPOLATION

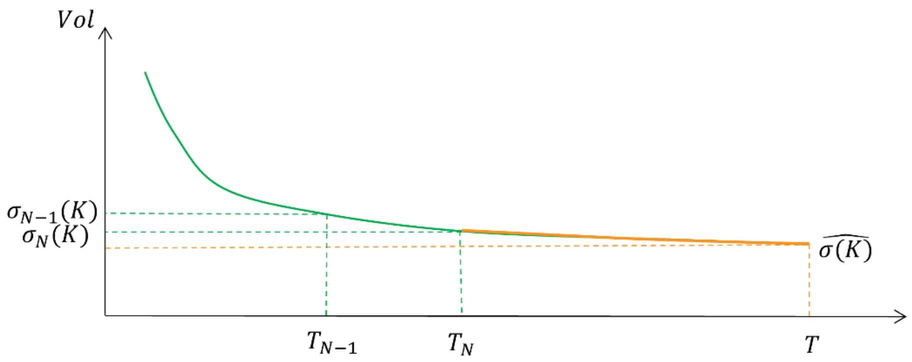
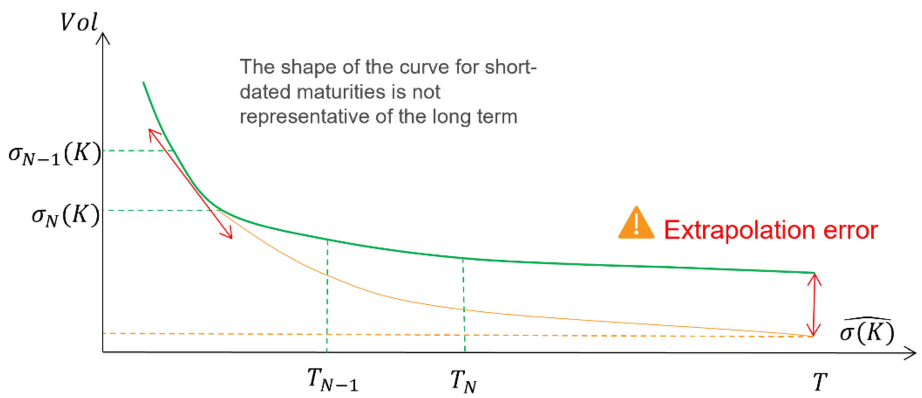


FIGURE 3: AN ILLUSTRATION OF EXTRAPOLATION ERROR



Because the quality of the extrapolation depends on the information contained in the initial sample, extrapolation errors may be observed when the shape of the curve is changing between the last available point and the extrapolated point (see illustration in Figure 3).

This issue is typically observed in practice when trying to extrapolate long-term data from listed market information where only a few maturities and strikes are liquid. For example, the table in Figure 4 provides the typical maximum maturity for listed markets used to build the volatility surfaces on some of the major indices.

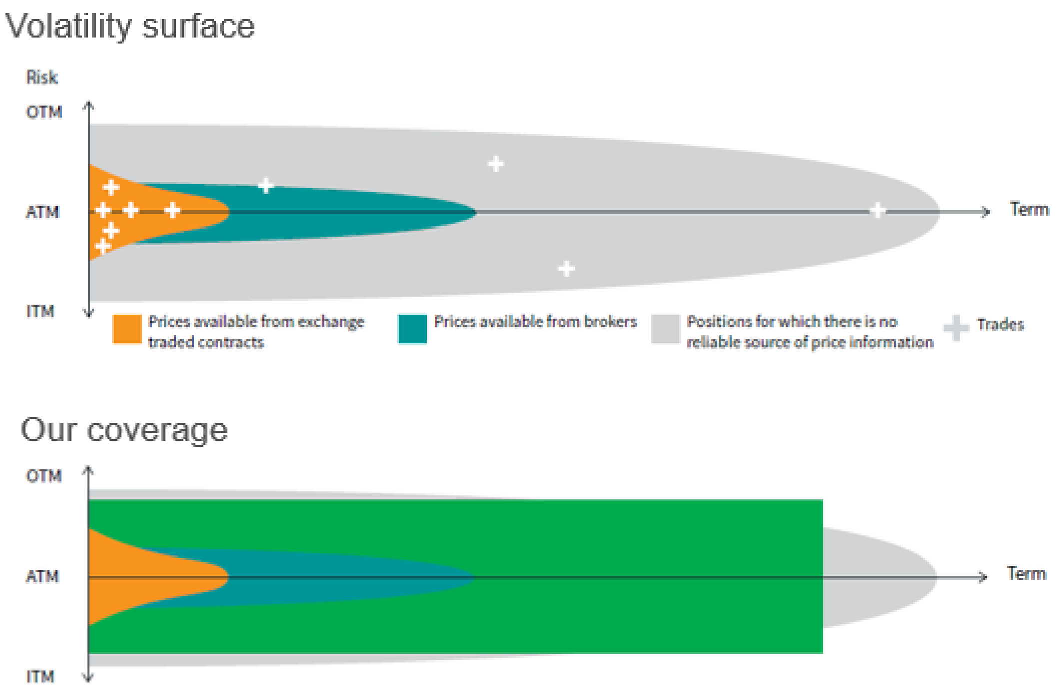
FIGURE 4: TYPICAL MAXIMUM LISTED MATURITY FOR SAMPLE INDICES USED BY IHS MARKIT (NOW A PART OF S&P GLOBAL)

INDEX	MAXIMUM LISTED MATURITY
EURO STOXX 50	4Y
STOXX:BANKS	2Y
SMI	18M
S&P 500	2Y
NIKKEI 225	6M
CAC 40	1Y

Introducing a new data source on long-term maturities

Even if not trading every day on the entire surface, market dealers have risk on pretty much all points of the surface and are therefore marking their books on the entire surface. IHS Markit (now a part of S&P Global) has access to a consensus of market dealer marks on derivatives through unique partnerships. It gives access to data on points where there is no information from listed markets or brokers, as illustrated in Figure 5.

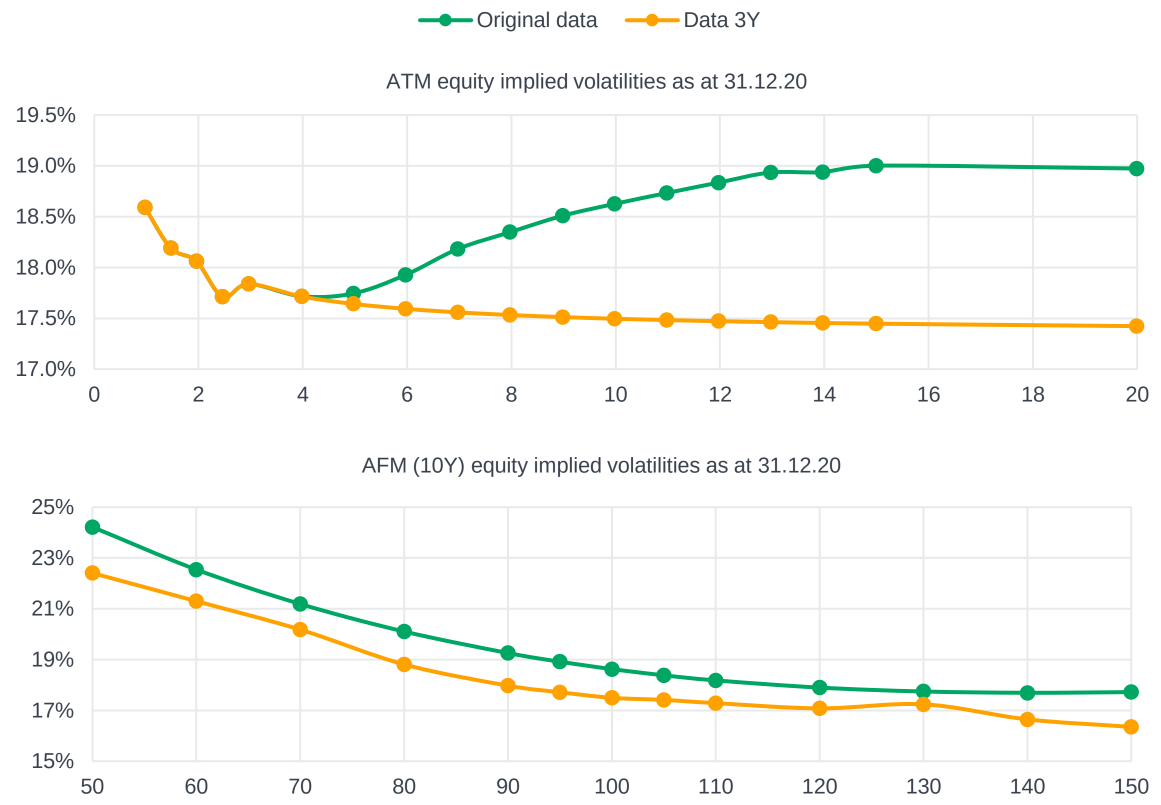
FIGURE 5: THE IHS MARKIT COVERAGE INTRODUCING NEW DATA SOURCES



IHS Markit (now a part of S&P Global) equity-implied volatility surfaces are then calibrated from a blend of exchange information and market-maker consensus information for best quality and coverage.

The graphs in Figure 6 compare IHS Markit (now a part of S&P Global) actual data on Euro Stoxx 50 and extrapolated data from maturity three years onwards as of 31 December 2020, such extrapolation being aligned with the approach described above and with the approach used by institutions only able to access listed market information.

FIGURE 6: COMPARISON OF IMPLIED VOLATILITY DATA



For ATM implied volatilities and a maturity of 20 years, the difference between the actual data and the extrapolated data is approximately 160 basis points (bps). For away-from-the-money (AFM) - i.e., ITM or OTM - implied volatility and a maturity of 10 years, the difference between the actual data and the extrapolated data is in the range of 100bps to 200bps, depending on the strike considered.

Impact analysis

In this last section, we present an impact analysis of the change in the data source in terms of solvency position for a typical French savings portfolio. The risk-neutral economic scenarios are generated by the economic scenario generator Milliman CHESS.[4] The comparison is based on the value of Solvency II Own Funds between:

- The use of short-term actual data up to maturity three years, with extrapolation up to maturity 20 years
- The use of the full data set up to maturity 20 years as provided by IHS Markit

The assessment is made as at two valuation dates (as at 31 December 2020, as per the example in the previous section, and as at 31 March 2021), with an objective to assess the stability of the difference between relying on long-term data as opposed to using the extrapolation method. The Euro Stoxx 50 index is projected based on two financial equity models:

- A Black-Scholes with time-dependent Deterministic Volatility (BSDV): This financial model relies on ATM-implied volatilities only.
- A SVJD: This financial model also takes into account additional ITM and OTM implied volatilities.

The results in terms of Own Funds calculation are presented in Figure 7.

[4] See https://www.milliman.com/en/products/milliman-chess.

FIGURE 7: IMPACT ASSESSMENT OF IMPLIED VOLATILITY DATA

DATA USED FOR BSDV CALIBRATION	RELATIVE VARIATION OF OWN FUNDS AS AT 31/12/2020	RELATIVE VARIATION OF OWN FUNDS AS AT 31/03/2021
Actual data up to 20Y	0%	0%
Actual data up to 3Y and extrapolation up to 20Y	+18%	+17%
Data used for SVJD calibration	Relative variation of Own Funds as at 31/12/2020	Relative variation of Own Funds as at 31/03/2021
Actual data up to 20Y	0%	0%
Actual data up to 3Y and extrapolation up to 20Y	+13%	+12%

For the case study considered, extrapolating actual implied volatility leads to an overestimation of the Own Funds, which is expected given that, for both valuation dates, the extrapolated approach leads to lower implied equity volatilities for medium-term to long-term tenors, and therefore an underestimated cost of options and guarantees for insurers. The conclusion remains the same when only ATM data are considered in the calibration process (though the BSDV model) or when ATM and AFM data are considered in the calibration process (through the SVJD model), although the difference is higher when only ATM data are considered. For this particular case study, an underestimation of the Solvency Capital Requirement (SCR) can also be observed when using extrapolated data as a result of two main factors:

- A higher loss-absorbing capacity of taxes and reserves (profit-sharing mechanism), as a result of overestimated Own Funds of the insurance company
- An underestimation of the lapse-down SCR, as the overall portfolio is more profitable based on extrapolated data

Beyond the spot impact of using the extrapolation methodology, we also notice that the misestimation of the Own Funds remains for the two valuation dates tested. However, such impacts could really depend on the shape of the extrapolated implied volatilities, and therefore the magnitude of the impact could change from one period to another. In particular, the impact can vary depending on the economic conditions and the long-term market expectations.

Concluding remarks

There has been an increasing interest in developing more robust and less volatile long-term implied volatility assumptions in recent years, driven primarily by new capital and financial regulations such as IFRS 17 and LDTI. There has been a number of model-driven attempts to address this challenge, including:

- Statistical approaches aimed at providing a better understanding of market expectations for more reasonable long-term implied volatilities for equity (including long-term mean reversion)[5]
- Refined modelling approaches of the volatility itself by considering its now acknowledged short memory and rough property, e.g., using the fractional Brownian motion[6]

However, a key ingredient to deriving long-term expectations is the data itself. In this context, our objective is to expand the depth of market information by considering all data observed within the OTC markets. As such, we expect the quality of data underlying economic scenario generators to become a central point of focus for business managers, regulators and auditors who aim to improve the accuracy of the valuation of long-term insurance liabilities for different capital and financial purposes.

[5] See http://faculty.mccombs.utexas.edu/carlos.carvalho/OnLongRunVariance.pdf and https://fr.milliman.com/fr-fr/insight/A-review-of-the-Solvency-II-equity-shock.

[6] See https://fr.milliman.com/fr-fr/insight/a-realistic-modelling-of-the-dynamics-of-equity-volatility and https://econpapers.repec.org/paper/aizlouvad/2021017.htm.

How can Milliman help?

Milliman has a depth of experience and expertise related to economic scenario generators, including the following services:

- Milliman CHESS. We offer a “one-size-fits-all” economic scenario generator (ESG) software-as-a-service (SaaS) solution with a large spectrum of best-in-class models. We provide advice on the best model in each context and the relevant tailored calibrations in line with each specific risk profile.
- Delivery of economic scenarios. We provide scenarios as a service, including custom calibrations and scenario generation at required frequencies
- Production support. Secure the process by outsourcing production work, with the assistance of our teams in the generation of economic scenarios during production campaigns.
- ESG review. Get updates and comfort on your in-house solution through a detailed external review of the modelling framework, including embedded parameters, implementation and validation of the results.
- ESG implementation support. We help you move to an in-house ESG implementation and get the relevant support: thorough choice of models and assumptions, technical specifications, functional specifications and core code implementation.
- Data quality for ESG software and services. We are working closely with IHS Markit (now a part of S&P Global) to provide the highest-quality data for all applications of economic scenarios for insurance and reinsurance valuation.

If you have any questions or comments on this paper or any other aspect of economic scenario generation, please contact your local Milliman consultants or the contact links below.

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