One-factor Hull-White Model Calibration for CVA
Part I: Instrument Selection With a Kink

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This paper is the first of a multi-part series on the calibration of the one-factor Hull-White short rate model for the purpose of computing CVAs (and xVAs) with an xVA system. It introduces an atypical bootstrapping scheme for the calibration of the short rate volatility. The second part focuses on the selection of the mean reversion parameter. In both expositions we present long-term time series results for EUR, JPY, and USD, covering the period from the beginning of 2009 (at the earliest) to spring 2020.

Uncollateralized netting sets or netting sets with weak credit support annexes can incur large valuation adjustment (xVA) charges due their impact on a dealer's counterparty credit risk exposure (CVA), need for funding (FVA and MVA), and capital constraints (KVA). Whereas the unadjusted value of derivatives is often determined and risk-managed with custom-tailored model and calibration approaches, valuation adjustments are netting set and higher-level metrics that require the joint modeling of risk factors across asset classes. The high dimensionality involved necessitates that xVAs are almost exclusively estimated using Monte Carlo simulation. To reduce the computational load of simulating an entire bank portfolio to many future exposure dates, it is common to use simpler models, trading excess accuracy for greater robustness, tractability, and speed.[1, 2]

Interest rate modeling provides a good example. Although a global model that can capture yield curve decorrelation and swaption volatility smile is ideal, many practical implementations use a single-factor Hull White (HW 1F) model.[3] The limited number of parameters inherent in the model means that it is unable to fit the entire ATM swaption volatility surface. Instead, a set of swaptions, with one length per expiry, is typically selected for calibration. Two frequent selections are

1. a coterminal (or diagonal) swaption set (Fig. 1 (a)) and
2. a columnar swaption set (Fig. 1 (b)).

For a single swap portfolio, coterminal swaptions matching the swap's maturity and struck at the swap's fixed rate are the ideal choice, providing smile and maturity aware xVAs. The columnar swaption set, on the other hand, may be preferred, e.g., when liquidity of the calibrating swaptions and long-datedness of the valuation are a particular concern. Both examples are local in nature as they typically revolve around individual instruments or portfolios.

If the objective is to *simultaneously* compute the xVAs of *multiple portfolios*, other ideas are required. For instance, in order to determine the xVAs of multiple single swap portfolios with various lengths reasonably well, a third calibration option, the focus of the paper, might be more appropriate:

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3. a chevron-shaped swaption set (Fig. 1 (c)).

Here the idea is to match the peak exposure of each of the swaps, which we assume appears half-way through the life of a swap and coincides with the trace of T/2-into-T/2 swaptions. The resulting upward leg is preempted half-way by a downward leg of coterminal swaptions that roughly match the maturities of the longest running swaps (e.g., 30 years in Fig. 1) (c)). Using this calibration, the HW 1F model can produce fairly reliable and accurate xVAs for at-the-money (ATM) swaps, in particular when the swap maturity distribution is relatively uniform or a priori unknown, as we demonstrate below. Calibration to coterminal or columnar swaptions, in contrast, tends to result in inferior and more variable xVA estimates, where the performance depends sensitively on the specifics of the swap maturity distribution.

To illustrate similarities and differences between the three instrument selection approaches in more detail, we simulate the CVAs of a series of at-the-money (ATM) single swap portfolios with 2, 5, 10, 15, 20, and 30-year maturities for the currencies EUR, JPY, and USD. The swaps are assumed to be transacted with fictitious lower-grade counterparties, and their CVAs are compared to two benchmarks:

- a smile and maturity aware CVA price that can be considered exact and is based on individual calibration for each swap as described above; and
- a multi-factor CVA price based on the four-factor Hull-White (HW 4F) model, which is calibrated to the entire ATM swaption surface.[3, 4]

The large number of calibration degrees of freedom of the HW 4F model means that it can fit the ATM swaption surface remarkably well and thus provides useful ATM CVA estimates. For all HW 1F simulations, the mean reversion parameter is an important extra degree of freedom, which we use to fit, in a least squares sense, the set of ATM instruments that are not selected for calibration. We expand on the role and impact of the mean reversion parameter in the second part of this two-part series.

Using these swaps, we perform monthly CVA calculations over the years 2009-2019 and weekly for 2020, and apply both benchmarks to compute root-mean-square relative error (RMSRE) metrics to quantify the typical CVA accuracy. The resulting CVA RMSRE time series are displayed in Figs. 2 and 3 and suggest that accuracy and consistency of the CVA calculations based on the different ATM instrument selection schemes can differ widely. The chevron-based CVAs, when judged by these metrics, are clearly preferable to those based on the alternative calibration approaches and are quite competitive with the results derived from the four-factor Hull-White model. The average relative CVA accuracy of the chevron approach is roughly in the 5%-range, with respect to either CVA benchmark, and typically stays below 10% even during more challenging times when market swaption volatilities and Hull-White models are less compatible with each other.[1] Closer inspection confirms that the main performance gain of the chevron-calibrated models originates from the CVAs of the shorter-termed swaps, with lengths of 2 and 5 years, see, e.g., the swap tenor resolved results in Tab. 1. Using any of the alternative calibration methods, this gain is lost as the resulting CVAs exhibit a larger variability across swap maturities and are typically harder to reconcile with the CVA benchmark values. The table also reinforces the notion that no one HW 1F instrument selection routine dominates all maturities. A model that is calibrated to instruments that reflect more specific portfolio characteristics can lead to more accurate, i.e. less model-dependent, results.

Probing the utility of the different HW 1F calibration approaches beyond ATM interest rate swaps, the quality of the CVA estimates can change drastically. For instance, the CVAs of in-the-money (ITM) swaps can be well estimated with any of the methods above as they are largely determined by the centers of the simulated swap rate distributions. Out-of-the money

[1] Occasionally, a satisfactory ATM swaption fit can only be achieved with a three or four-factor Hull White model.

(OTM) swaps in contrast can lead to large relative CVA discrepancies, mostly due to their greater sensitivity to swap rate distribution tails, or implied volatility smiles, and due to the naturally much smaller CVA values. The last aspect, of course, also suggests that these CVAs may be negligible in comparison to other CVA and xVA charges.

Our findings also appear to be largely in line with Totem's xVA consensus pricing results, where the CVA submissions for ITM swaps consistently exhibit less variation than those for ATM and OTM swaps. Such behavior likely reflects the stronger impact of the idiosyncratic valuation approaches taken by the participating banks and is mimicked here to some extent by the different calibration methods.

Finally, considering the recent economic turmoil triggered by the global coronavirus health crisis, the chevron-based calibration approach continues to hold up well and to provide relatively reliable CVA numbers. During the tensed days in March 2020, the ATM swaption volatility surfaces of all three currencies took on extreme shapes at short swaption maturities, rising sharply when moving from shorter to longer swaption lengths. Under such conditions, the models calibrated to the coterminal swaptions, for instance, can lead to overestimated exposures for shorter-termed swaps (see CVA RMSRE curves for EUR and USD in 2020; JPY appears to be an exception, likely due to more pronounced volatility smiles or skews), exacerbating the CVA hit caused by the generally widening counterparty credit spreads. As of mid-April 2020, the extreme features of the swaption volatility surfaces for EUR, JPY, and USD have again weakened somewhat. All in all, it is very plausible that the currently unfolding macro shock will drive counterparty credit risk and, more generally, credit risk, in new directions.

References

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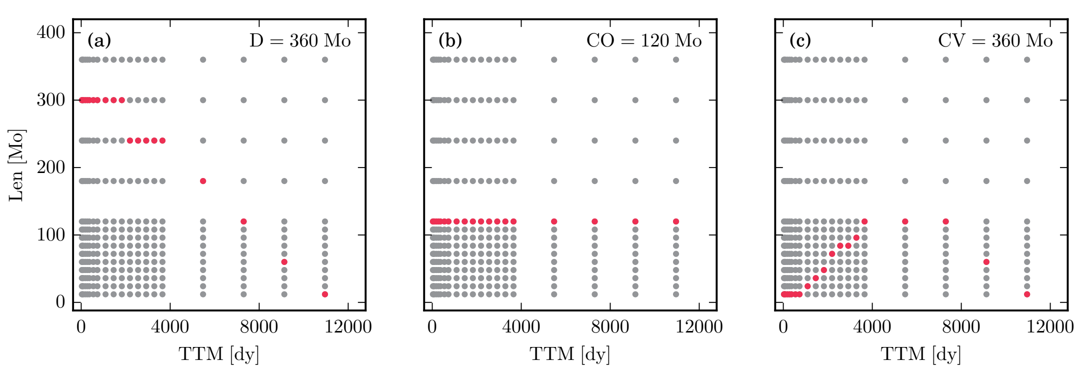

Figure 1: Sample HW 1F bootstrap calibration patterns in the ATM implied swaption volatility matrix: (a) 30-year swaption diagonal (or coterminal), (b) 10-year swaption column, and (c) 30-year swaption chevron (i.e. the chevron base spans 30-years).

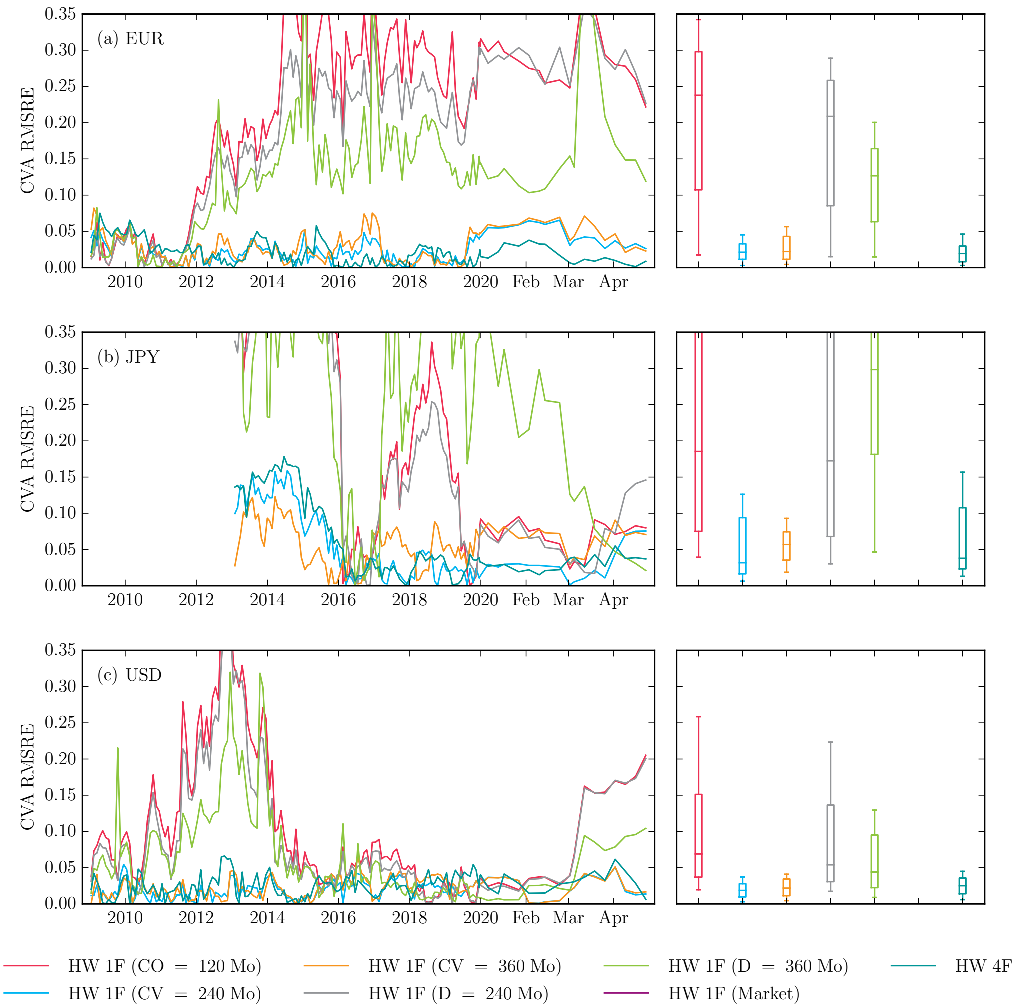

Figure 2: Monthly (2009-2019) and weekly (2020) sampled swap CVA RMSREs for different HW 1F calibration schemes (where CV = chevron, CO = column, D = diagonal/coterminal) and the HW 4F model. The RMSREs are computed with respect to the maturity and smile-aware CVA benchmark (HW 1F (Market); its graphs are trivially zero). The box plots show a collapsed view of the time series and indicate 10, 25, 50, 75, and 90-percentiles. The time series extend up to 23 April 2020. For more details, see the main text and Tab. 1.

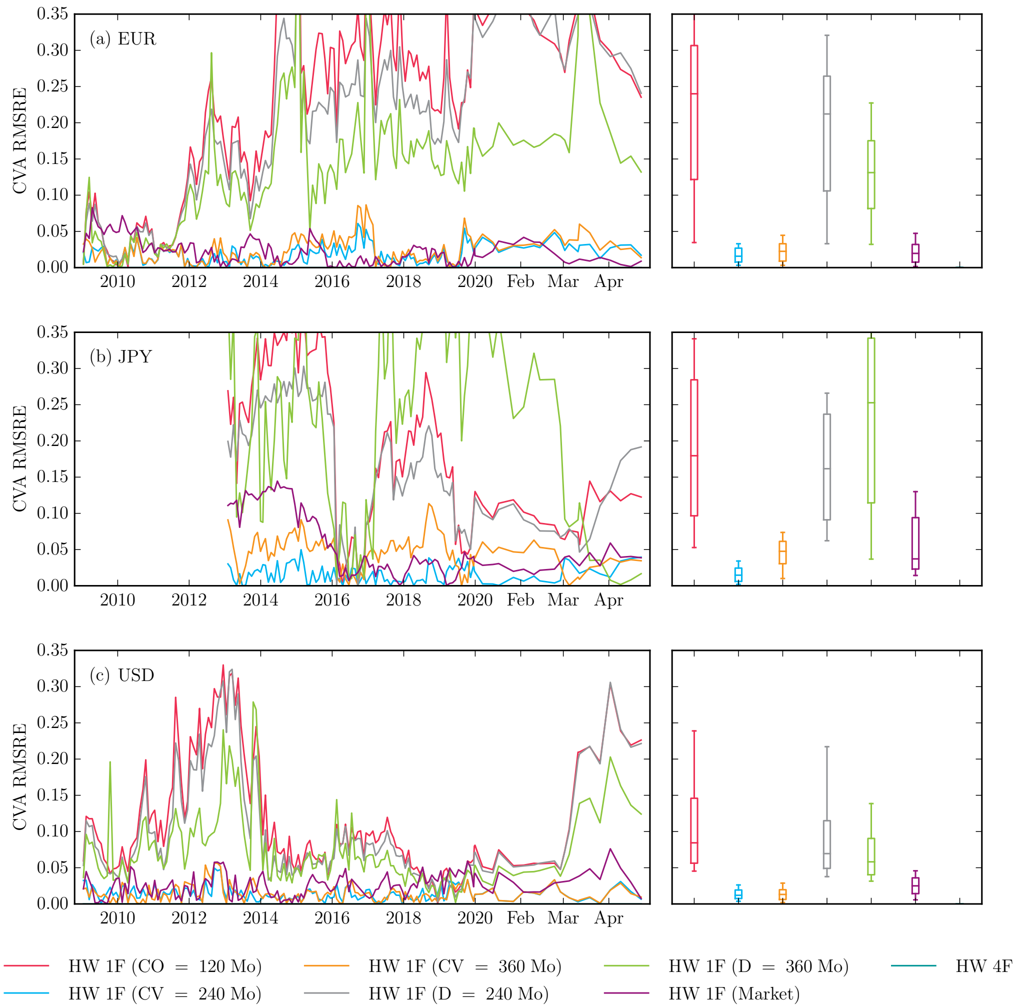

Figure 3: Similar to Fig. 2 but with CVA RMSREs referencing the HW 4F CVA benchmark. See caption of Fig. 2, and Tab. 1 and the main text for more details. (The graphs for HW 4F are trivially zero.)

	Swap Maturity [yr]	HW 1F CO = 120 Mo	CV = 240 Mo	CV = 360 Mo	D = 240 Mo	D = 360 Mo
(a)						
EUR	2	79 (114)	4 (5)	4 (5)	71 (103)	77 (98)
	5	18 (27)	3 (5)	3 (5)	10 (17)	9 (12)
	10	4 (5)	4 (5)	4 (6)	2 (2)	6 (7)
	15	2 (3)	3 (6)	4 (7)	2 (3)	9 (8)
	20	3 (4)	3 (3)	3 (3)	3 (3)	7 (5)
	30	6 (6)	11 (10)	8 (7)	11 (15)	2 (2)
JPY	2	84 (96)	3 (3)	3 (3)	73 (73)	102 (57)
	5	44 (48)	4 (5)	4 (5)	29 (30)	43 (31)
	10	16 (18)	4 (4)	4 (5)	7 (8)	10 (19)
	15	9 (12)	5 (5)	5 (5)	6 (8)	7 (11)
	20	8 (10)	7 (9)	6 (6)	11 (14)	6 (8)
	30	10 (11)	21 (15)	12 (10)	24 (25)	7 (8)
USD	2	37 (50)	4 (5)	4 (4)	37 (44)	40 (48)
	5	6 (11)	4 (4)	4 (4)	5 (8)	4 (8)
	10	3 (3)	5 (3)	5 (3)	5 (3)	6 (4)
	15	3 (3)	3 (3)	4 (3)	4 (2)	7 (4)
	20	2 (2)	2 (2)	2 (2)	2 (2)	5 (3)
	30	2 (3)	3 (4)	3 (3)	3 (3)	3 (2)
(b)						
EUR	2	89 (125)	2 (2)	2 (2)	88 (109)	86 (111)
	5	17 (31)	3 (4)	3 (4)	10 (21)	9 (15)
	10	3 (3)	6 (7)	6 (7)	4 (4)	8 (10)
	15	1 (2)	4 (6)	4 (7)	3 (3)	10 (13)
	20	1 (2)	2 (2)	2 (3)	1 (2)	8 (10)
	30	7 (4)	11 (6)	8 (5)	11 (11)	1 (2)
JPY	2	83 (86)	6 (5)	6 (5)	73 (62)	99 (55)
	5	41 (34)	2 (3)	2 (3)	25 (20)	36 (39)
	10	6 (8)	8 (9)	8 (9)	6 (4)	12 (12)
	15	2 (2)	7 (6)	9 (7)	4 (4)	12 (10)
	20	2 (2)	2 (2)	7 (5)	2 (2)	8 (7)
	30	4 (5)	16 (9)	8 (6)	19 (20)	3 (3)
USD	2	39 (38)	5 (5)	4 (4)	38 (30)	42 (37)
	5	5 (9)	4 (3)	4 (4)	4 (8)	4 (5)
	10	2 (2)	3 (4)	3 (4)	3 (3)	4 (5)
	15	1 (1)	2 (2)	2 (3)	2 (3)	5 (5)
	20	1 (1)	1 (1)	1 (2)	1 (1)	4 (3)
	30	3 (5)	6 (5)	5 (4)	4 (6)	1 (1)

Table 1: Medians and interquartile ranges (in parentheses) of the CVA RMSRE time series (in percent) resolved by swap maturity, revealing the strengths and weaknesses of the different HW 1F calibration approaches in detail. The medians and interquartile ranges are computed for sub-portfolios of fixed maturity, consisting of one receiver and one payer swap each, and are defined similarly to those displayed by the boxplots of Figs. 2 and 3. The CVA benchmark in (a) is the maturity and smile-aware CVA benchmark (HW 1F (Market)) and in (b) the HW 4F CVA benchmark.